Educational Computer Simulations for Visualizing and Understanding the Interaction of Electromagnetic Waves with Metamaterials.

Ana Grande, Oscar González, José A. Pereda and Ángel Vegas
Dpto. Ingeniería de Comunicaciones
Universidad de Cantabria,
Santander, Spain

Abstract— **In contrast to ordinary materials, metamaterials show interesting properties that are not found in nature. These properties are counterintuitive and difficult to understand for the students. This work describes a series of virtual experiments that were carried out by means of an extension of the FDTD technique for modeling metamaterials. The result of the simulations was recorded and a collection of videos developed as a supplementary tool for teaching at advanced undergraduate and graduate levels. The objective is to help the student to understand the interaction of electromagnetic waves with these media by means of the visualization of the evolution in time of the physical phenomena.**

Keywords-component; educational software, visualization, metamaterials

I. INTRODUCTION

Over the last decade a great deal of attention has been paid to a new kind of media named "metamaterials". Metamaterials are artificially structured composite media with a unique electromagnetic response not present in naturally occurring materials [1].

The nonconventional properties of metamaterials were theoretically investigated by Veselago in 1968 [2], and stem from the fact that these media present simultaneously negative permittivity ε and permeability μ. However, it was in 1998 when the first possible realization of a metamaterial was achieved by the appropriate combination of metallic wires and split-ring resonators [3]–[4].

Since this first synthesis there has been an increasing interest on these media and nowadays, metamaterials are a relevant hot topic in science and technology becoming one of the fastest moving research fronts according to the Web of Science [5]. The keen interest on this kind of media is due to their exotic electrodynamic properties, such as the negative refractive index ($n < 0$), backward-wave propagation, negative Goos-Hänchen shift, reversal Doppler effect, inverse Cerenkov radiation,... and also for their important potential applications in the development of novel devices [6], perfect lenses [7], invisibility [8], [9], etc.

In general, students have difficulty in perceiving the behavior of electromagnetic waves. The situation is worsened when metamaterials are involved, since they exhibit an electromagnetic response not found in natural media that seems counterintuitive to the students.

Electromagnetic simulators have proven to be useful teaching/learning tools to provide physical insight into the phenomena under study. This goal is more easily achieved by time-domain simulators, since they emulate the progression of the fields as they actually evolve in space and time [10]-[12].

The finite-difference time-domain (FDTD) is an accurate numerical technique for solving Maxwell's equations. The conventional FDTD scheme has been successfully extended to incorporate the modeling of a broad range of electromagnetic simulation problems, what has made the FDTD method one of the most powerful and flexible numerical techniques that enables the solution of problems in the time domain [13].

In this work an extension of the FDTD technique for modeling metamaterials [14]-[15] has been used to carry out a series of virtual experiments. These experiments have been carefully chosen to illustrate the characteristic behavior of electromagnetic waves in these media. The result of the simulations has been recorded and a collection of videos has been created. The videos are intended for advanced undergraduate and graduate students and their objective is to promote understanding by means of the visualization of the evolution in time of the fields in metamaterials.

II. FDTD MODELING OF METAMATERIALS

When simulating the propagation of electromagnetic waves the starting point are Maxwell's curl equations. For two dimensional problems and for the TE_z polarization case, these equations can be written in the Laplace domain as

$$\begin{aligned} sD_x &= \frac{\partial H_z}{\partial y}, \\ sD_y &= -\frac{\partial H_z}{\partial y}, \\ sB_z &= \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}. \end{aligned} \quad (1)$$

In the Laplace domain, the constitutive relationships that characterize the metamaterials are given by:

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$$\vec{D}(s) = \varepsilon(s)\vec{E}(s),$$
$$\vec{B}(s) = \mu(s)\vec{H}(s), \quad (2)$$

where ε is the electric permittivity and μ the magnetic permeability of the medium.

Any physical realization of a metamaterial is dispersive, i.e., its effective constitutive parameters depend on the frequency. In those frequency regions where the permittivity and permeability present simultaneously negative values the medium will have a negative refractive index and will exhibit the interesting physical properties of metamaterials.

In the classical FDTD approach [13] the space and time derivatives in Maxwell's curl equations are approximated by means of second-order central differences and the computational space is divided into cells where the electric and magnetic field components are alternately distributed. This leads to a system of explicit finite-difference equations that allows us to iteratively compute the electric and the magnetic fields. The resulting algorithm is a marching-in-time procedure that simulates the actual electromagnetic waves in a finite spatial region.

The original FDTD scheme has been successfully extended to the modeling of different kinds of materials. In [14] an approach to incorporate metamaterials was presented.

In this work that extension of the FDTD technique has been used to visualize the most typical phenomena of wave propagation in these media. In particular we have considered two dimensional problems for the TE_z polarization case. The computational domain was surrounded by PMLs absorbing boundary conditions.

In our simulations, for the sake of simplicity, and in order to have a wide double negative frequency band, we have assumed that the permittivity and the permeability follow identical Drude dispersion models, which in the Laplace domain can be expressed as:

$$\varepsilon_r(s) = \mu_r(s) = 1 + \frac{\omega_p^2}{s^2 + s\omega_c} \quad (3)$$

where ω_p and ω_c are the plasma and the collision frequencies respectively. According to (3), in the lossless case (ω_c= 0), for frequencies $\omega < \omega_p$ both constitutive parameters will be negative and the medium will present interesting electromagnetic properties.

III. Visualizing and Understanding Physical Phenomena in Metamaterials

A. *Negative refraction*

According to Snell's law, when a wave impinges at the interface between two media the relationship between the angles of incidence and refraction is given by:

$$n_1 \sin\theta_{inc} = n_2 \sin\theta_{ref}. \quad (4)$$

Natural media present positive refractive indices ($n > 0$), and therefore positive refraction angles. However, metamaterials exhibit a negative refractive index ($n < 0$). As a result, Snell's law is reversed at the interface between ordinary media and metamaterials. To display the anomalous refraction to the student we have designed two numerical experiments and generated the corresponding videos. In the first one we can visualize the incidence of a CW Gaussian beam on a medium with ($n_2 > 0$). As expected, the beam is refracted with a positive angle. Fig. 1 shows a snapshot of the simulation.

In the second simulation the same Gaussiam beam travelling in air impinges on a metamaterial with ($n_2 < 0$). By means of the recorded movie the student will be able to visualize how refraction is reversed, i.e., the refracted beam is on the same side of the normal as the incident beam is.

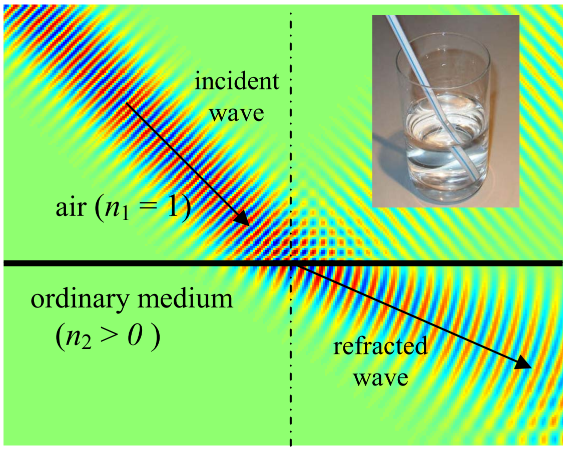


Figure 1. Positive refraction at the interface between air and an ordinary medium

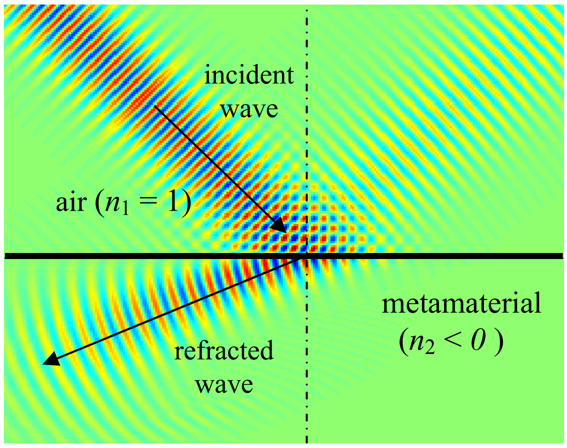


Figure 2. Negative refraction at the interface between air and a metamaterial: the light bends back from the normal.

B. *Backward propagation*

The phase velocity can be written as a function of the refractive index as

$$v_{ph} = \frac{c}{n} \quad (5)$$

where c is the speed of light in vacuum. Thus, due to the negative refractive index, metamaterials present negative phase velocity. However, the direction of the time-averaged flux of energy is determined by the real part of the Poynting vector

$$\vec{S} = \frac{1}{2}\vec{\mathrm{E}}\times\vec{\mathrm{H}}^{*} \qquad (6)$$

which is unaffected by a simultaneous change of sign of ε and μ. Therefore, in a metamaterial the energy and the wavefront travel in opposite directions (backward propagation).

For most students, it is difficult to mentally visualize the backward propagation phenomenon, since it is counterintuitive that the energy and the wavefront travel in opposite directions.

To promote the understanding of this phenomenon we have simulated the normal incidence of a wave on a metamaterial slab. Fig. 3 shows the snapshots of field propagating across a slab at two different time instants. In the figure we can see how in the metamaterial region the wavefront travels in the negative x-direction, while the power is flowing in positive x-direction, since the signal is transmitted into the second air region.

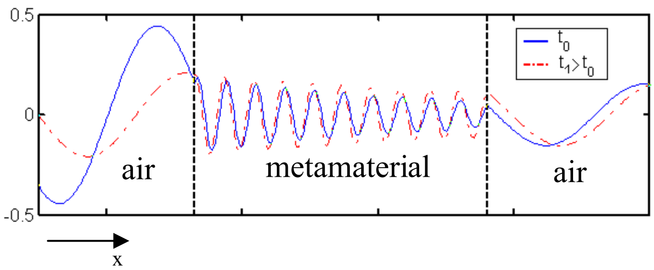


Figure 3. Backward propagation, in the metamaterial the power flows in the positive x-direction while the wavefront travel in the negative x-direction.

A static image like Fig. 3 does not clearly illustrate the backward propagation in the metamaterial slab. However, by means of the simulation movie the students will be able to visualize the evolution of the fields in time, which will help them to understand this phenomenon.

C. *Pendry's lens*

Another remarkable property of metamaterials is the possibility of creating flat lenses. Materials with negative refractive index refract light at a negative angle which allows a flat lens to bring waves into focus, whereas media with $n > 0$ always require curved surfaces to focus light. In fact a flat lens made of a metamaterial with $n = -1$ and thickness d, will produce two foci, one inside the slab and the other outside the slab. As can be seen in Fig. 4, for a source located at a distance d_0 from the front face of the slab the foci will appear at $d_{f1}=|d_0|$ and $d_{f2} = 2d\text{-}|d_0|$ [16].

In general, lenses introduce a phase correction to each of the Fourier components so that at some distance beyond the lens the fields focus. However for large values of the transverse wave vector these waves become evanescent and their amplitude decays exponentially so no phase correction will restore their original amplitude. As a result, for ordinary lenses, the evanescent waves are removed from the image.

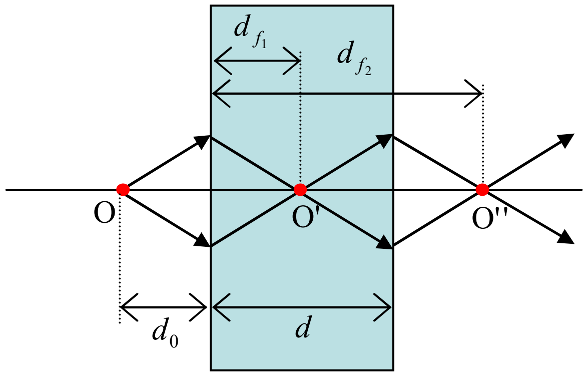


Figure 4. Flat lens: a slab made of a metamaterials with $n = -1$ focuses the waves producing two foci, one inside the slab and the other one outside

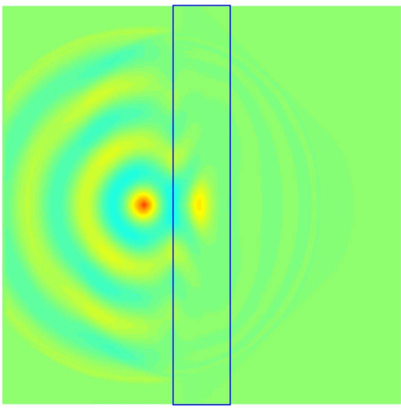
In the year 2000 Pendry [7] pointed out one of the most promising properties of a metamaterial slab. In that work Pendry theoretically demonstrated that a lens made of a metamaterial slab with $n = -1$ could restore the amplitude of evanescent waves, which may enable the development of "super-lenses" with a focusing resolution beyond the diffraction limit.

In order to show the student the behavior of Pendry's lens we have simulated a 10-mm thick slab of a metamaterial. A source is located 5 mm in front of the slab. The working frequency is f_0 = 30 GHz. At that frequency, the refraction index of the slab is $n(f_0) = -1 - j0.0011$. In Fig. 5 we can see three snapshots of the simulation. In the first one we clearly distinguish the location of the source. In the second one we can see how the first focus begins to emerge in the middle of the slab. Finally in the third image the second focus appears 5 mm behind the slab. In this last image we can also distinguish some surface waves at the first interface of the slab.

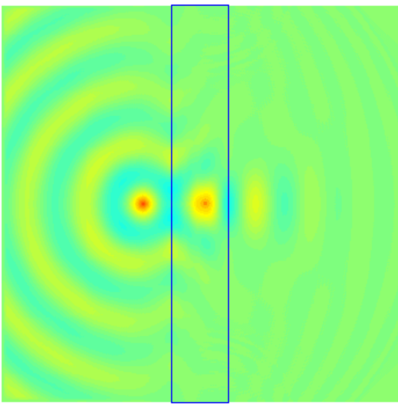
Although in Fig. 5 we can perceive the behavior of the fields in Pendry's perfect lens, it is indeed much more clearly visible when the time-domain solution is displayed. Thus, watching the simulation video the students will be able to see not only the formation of the two images by a flat lens but they will also be able to watch the dynamics of the surface waves, how they are built up and how they travel along the interfaces. It is of considerable interest that the students visualize and understand how these surface modes propagate since they are responsible for the amplification of the evanescent waves in a metamaterial slab.

D. *Negative Goos-Hänchen shift*

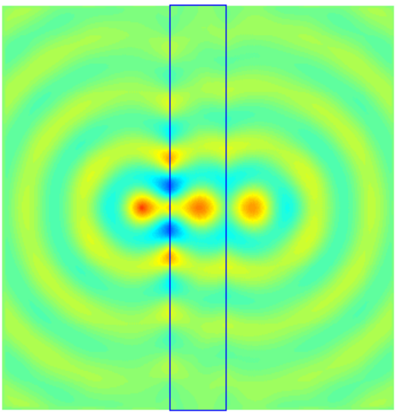
When a beam of light is totally reflected from a plane interface between two dielectrics, there is a lateral shift between the reflected light beam and the incident one. This lateral shift was predicted by Newton and experimentally demonstrated by Goos-Hänchen. However, when the beam is reflected from a metamaterial this phenomenon is reversed and the beam undergoes a negative displacement [17]. This phenomenon is schematically depicted in Fig. 6.



a)



b)



c)

Figure 5. Pendry-s lens: a) the source is located 5 mm in front of the slab, b) the first focus appears in the middle of the slab, c) the second focus emerge and surface waves travel along the interface.

To promote the understanding of this reversal phenomenon we have run two simulations and recorded the corresponding videos. The first video displays the propagation of a wave in a medium with n_1 = 9 that impinges with an incidence angle of 45º on a medium with n_2 = 3. In this situation the critical angle equals 35.26º. Therefore, total reflection will take place and the reflected beam will suffer a positive shift ($\Delta > 0$). Fig. 7 shows one of the last snapshots of the movie where we can see the positive Goos-Hänchen effect.

The goal of the second video is to show the students how this shift is reversed when metamaterials are involved. Thus if the medium 2 is substituted by a metamaterial with n_2 = -3, and we run the simulation in an analogous way we find that the totally reflected beam undergoes a negative lateral displacement ($\Delta < 0$). This phenomenon can be seen in Fig. 8.

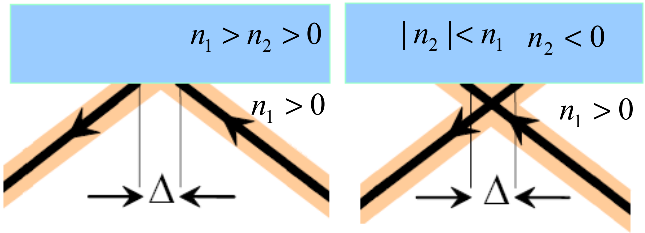


Figure 6. Goos-Hänchen effect: a) positive shift b) negative shift.

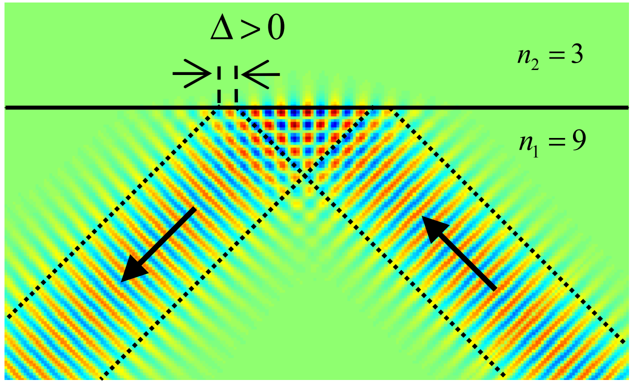


Figure 7. Positive Goos-Hänchen shift ($\Delta > 0$) at the interface between two ordinary media.

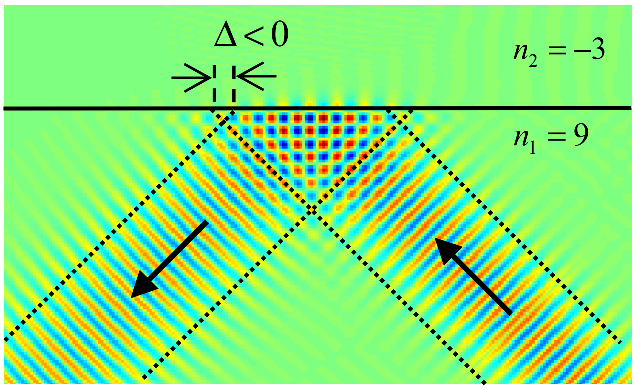


Figure 8. Negative Goos-Hänchen shift ($\Delta < 0$) at the interface of an ordinary medium and a metamaterial.

CONCLUSION

In this work an extension of the FDTD technique for modeling metamaterials has been used to carry out a series of virtual experiments. The result of the simulations has been recorded and a collection of videos has been created.

These videos provide aid in the teaching of the exotic electromagnetic properties of metamaterials, such as negative refraction, backward propagation, negative Goos-Hänchen shift, etc. By means of the visualization of the progression of the fields as they actually evolve in space and time the students will acquire a deeper physical insight into wave propagation in this kind of artificial materials.

REFERENCES

[1] D. R. Smith, et al, "Metamaterials and negative refractive index", Science 305, 788 (2004).

[2] V. G. Veselago, "The electrodynamics of substances with simultaneously negative values of ε and μ," Soviet Physics Uspekhi., vol. 10, no. 4, pp. 509–514, 1968.

[3] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Lowfrequency plasmons in thin wire structures," J. Phys.: Condens. Matter, vol. 10, pp. 4785–4809, 1998.

[4] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," IEEE Trans. Microw. Theory Tech., vol. 47, no. 11, pp. 2075–2081, Nov. 1999

[5] http://www.esi-topics.com/fmf/maps/september2005-map.html

[6] R. Marqués, F. Martín, M. Sorolla, Metamaterials with negative parameters.: Theory, Design, and Microwave Applications, Wiley-Interscience, 2009.

[7] J. B. Pendry, "Negative refraction makes a perfect lens," Phys. Rev. Lett., vol. 85, no. 18, pp. 3966–3969, Oct. 2000.

[8] J. B. Pendry, D. Schurig, D. R Smith, "Controlling electromagnetic fields", Science 312, 1780, 2006.

[9] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, D. R. Smith1, "Metamaterial electromagnetic cloak at microwave frequencies", Science 314, 977, 2006.

[10] A. W. Glisson, A. Z. Elsherbeni, "An interactive 1D matlab FDTD code for education," Computer Applications in Engineering Education, vol. 9, no. 2, pp. 136-147, 2001.

[11] D. W. Ward, K. A. Nelson, "Finite-difference time-domain (FDTD) simulations of electromagnetic wave propagation using a spreadsheet, " Computer Applications in Engineering Education, vol. 13, no. 3, pp. 213-221, 2005.

[12] G. Cakir, M. Cakir, L. Sevgi, "A multipurpose FDTD-based two-dimensional electromagnetic virtual tool," IEEE Antennas and Propagation Magazine, vol. 48, no. 4, pp. 142-151, 2006.

[13] A. Taflove, S. Hagness, Computational electrodynamics: the finite-difference time-domain method, Artech House, Inc, 2005.

[14] A. Grande, J. A. Pereda, O. González and A. Vegas, "Stability and accuracy of a Finite-Difference Time-Domain scheme for modeling Double-Negative media with high-order rational constitutive parameters, " IEEE Trans. Microwave Theory and Tech., vol. 56, no. 1, pp. 94-104, Jan. 2008.

[15] A. Grande, J. A. Pereda, O. González, A. Vegas, "On the equivalence of several FDTD formulations for modeling electromagnetic wave propagation in double-negative metamaterials," IEEE Antennas and Wireless Propagat. Lett., vol.6, no.1, pp. 324-327, Dec. 2007

[16] R. W. Ziolkowski y E. Heyman, "Wave propagation in media having negative permittivity and permeability," Phys. Rev. E, vol. 64, no. 5, 056625, Oct. 2001.

[17] P. R. Berman, "Goos-Hänchen shift in negatively refractive media," Phys. Rev. E, vol. 66, 067603-1, Dec. 2002.